Correction of misalignment errors in stereoscopic PIV systems

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OUTLINE

• Introduction on Stereo PIV technique

• Correction of formulae to compute viewing angles

• Correction of misalignment errors

• Experimental results

• Conclusions
STEREOPIV PROCEDURE WITH GEOMETRIC RECONSTRUCTION

- Achievement of Scheimpflug condition
- Calibration
- Storage of images
- Mapping function
- Dewarping of images
- Iterative Deformation PIV Process
- Data analysis
- Geometric reconstruction of three-dimensional flow field
CALIBRATION

An interpolating function from object to image coordinates is used:

\[ \bar{X} = \bar{F}(\bar{x}) \]

With:

\[
\bar{X} = \begin{pmatrix} X^1 \\ Y^1 \\ X^2 \\ Y^2 \end{pmatrix}, \quad \bar{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}
\]

Coefficients of mapping function are calculated with the least squares method.
**CALIBRATION WITH CAMERA PINHOLE MODEL**

The model consists of 6 *extrinsic* parameters which describe the camera pinhole orientation and position in the object space, and of 6 *intrinsic* parameters, which are specific to the camera itself:

- $s_x$ pixel aspect ratio
- $k_1$ and $k_2$ radial distortion factors (first and second order)
- $f$ focal length
- $(u_0, v_0)$ intersection of the optical axis with the image plane
CALIBRATION: DIFFERENT INTERPOLATING FUNCTIONS USED

In this work, following interpolating models have been analysed:

\[ \hat{F}(x) = a_0 + a_1 x + a_2 y + a_3 z + a_4 x^2 + a_5 xy + a_6 y^2 + a_7 xz + a_8 yz + a_9 z^2 + a_{10} x^3 + \]
\[ a_{11} x^2 y + a_{12} xy^2 + a_{13} y^3 + a_{14} x^2 z + a_{15} xyz + a_{16} y^2 z + a_{17} xz^2 + a_{18} yz^2 \]

\[ \hat{F}(x) = a_0 + a_1 x + a_2 y + a_3 z + a_4 x^2 + a_5 xy + a_6 y^2 + a_7 xz + a_8 yz + a_9 z^2 + a_{10} x^3 + \]
\[ a_{11} x^2 y + a_{12} xy^2 + a_{13} y^3 + a_{14} x^2 z + a_{15} xyz + a_{16} y^2 z + a_{17} xz^2 + a_{18} yz^2 + a_{19} z^3 \]

\[ \hat{F}(x) = \frac{a_0 + a_1 x + a_2 y + a_3 z + a_4 x^2 + a_5 xy + a_6 y^2 + a_7 xz + a_8 yz + a_9 z^2}{1 + a_{10} \dot{x} + a_{11} \dot{y} + a_{12} \dot{z} + a_{13} x^2 + a_{14} xy + a_{15} y^2 + a_{16} xz + a_{17} yz + a_{18} z^2} \]

\[ \hat{F}(x) = \frac{a_0 + a_1 x + a_2 y + a_3 z}{1 + a_4 x + a_5 y + a_6 z} \]
RECONSTRUCTION OF FLOW FIELD

Stereo PIV Technique
RECONSTRUCTION OF FLOW FIELD
RECONSTRUCTION OF FLOW FIELD

\[ \begin{align*}
\Delta x &= \frac{\Delta x_1 \tan \alpha_2 - \Delta x_2 \tan \alpha_1}{\tan \alpha_2 - \tan \alpha_1} \\
\Delta z &= \frac{\Delta x_1 - \Delta x_2}{\tan \alpha_2 - \tan \alpha_1} = \frac{\Delta y_1 - \Delta y_2}{\tan \beta_2 - \tan \beta_1} \\
\Delta y &= \frac{\Delta y_1 \tan \beta_2 - \Delta y_2 \tan \beta_1}{\tan \beta_2 - \tan \beta_1} = \\
&= \frac{\Delta y_1 + \Delta y_2}{2} + \frac{\Delta z}{2} (\tan \beta_1 + \tan \beta_2)
\end{align*} \]

With:

\[ \begin{align*}
\tan(\alpha_{1,2}) &= \frac{\Delta x}{\Delta z} = \frac{X_{1,2}^z}{X_{1,2}^x} \\
\tan(\beta_{1,2}) &= \frac{\Delta y}{\Delta z} = \frac{Y_{1,2}^z}{Y_{1,2}^y}
\end{align*} \]
MISALIGNMENT ERRORS

Camera #1

Camera #2

Calibration plane

Measurement plane

P

P_1

P_2

Disparity Vector

Correction of misalignment errors

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Achievement of Scheimpflug condition → Calibration → Storage of images

Mapping function → Dewarping of images → Disparity map → Computation of measurement plane

Iterative Deformation PIV Process with inner dewarping of images → Geometric reconstruction of three-dimensional flow field → Data analysis

**STEREOPIV PROCEDURE PROPOSED TO CORRECT MISALIGNMENT ERRORS**
EXPERIMENTAL RESULTS

The same pattern has been used to perform the calibration and to simulate the flow field.
Experimental results

### Results of calibration with 3 planes using CPM, P332, R222, R11

<table>
<thead>
<tr>
<th>Interpolating method</th>
<th>RMS Cam 0 - Cam 1</th>
<th>MAX ERROR Cam 0 - Cam 1</th>
<th>R-SQUARE Cam 0 - Cam 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPM</td>
<td>0.3992</td>
<td>1.374</td>
<td></td>
</tr>
<tr>
<td>P332</td>
<td>0.2924</td>
<td>1.038</td>
<td>0.9992</td>
</tr>
<tr>
<td>R222</td>
<td>0.2942</td>
<td>1.014</td>
<td>0.9993</td>
</tr>
<tr>
<td>R111</td>
<td>0.3865</td>
<td>1.430</td>
<td>0.9952</td>
</tr>
</tbody>
</table>

### Results of calibration with 5 planes using CPM, P332, P333, R222, R11

<table>
<thead>
<tr>
<th>Interpolating method</th>
<th>RMS Cam 0 - Cam 1</th>
<th>MAX ERROR Cam 0 - Cam 1</th>
<th>R-SQUARE Cam 0 - Cam 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPM</td>
<td>0.3954</td>
<td>1.377</td>
<td>0.9981</td>
</tr>
<tr>
<td>P332</td>
<td>0.29291</td>
<td>1.0477</td>
<td>0.99818</td>
</tr>
<tr>
<td>P333</td>
<td>0.29233</td>
<td>1.0453</td>
<td>0.99834</td>
</tr>
<tr>
<td>R222</td>
<td>0.29464</td>
<td>1.0866</td>
<td>0.99792</td>
</tr>
<tr>
<td>R111</td>
<td>0.38584</td>
<td>1.4197</td>
<td>0.99701</td>
</tr>
</tbody>
</table>
Uniform flow field along $z$ ($w = 1 \text{ mm}$) without misalignment: correction of the formulae to compute viewing angles

Experimental results

First order formulae

Second order formulae

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Uniform flow field along $z$ ($w = 1\text{mm}$) without misalignment: correction of the formulae to compute viewing angles

**Experimental results**

### v component of flow field

#### First order formulae

#### Second order formulae
Experimental results

Uniform flow field along $z (w = 1 \text{mm})$ without misalignment:
correction of the formulae to compute viewing angles

**w component of flow field**

First order formulae

Second order formulae
Uniform flow field along x \( (u'=1\,mm) \) with angular misalignment \( \alpha=-1^\circ \): correction of flow field using the disparity map

\[
z = 9.20e-002 - 1.69e-002 x + 9.51e-006 \cdot y
\]

Equation of measurement plane obtained by means of disparity map

<table>
<thead>
<tr>
<th>Prediction:</th>
<th>( u = 0.986,mm )</th>
<th>( v = 0,mm )</th>
<th>( w = -0.0169,mm )</th>
</tr>
</thead>
</table>

**Results with disparity map**

<table>
<thead>
<tr>
<th>Statistics</th>
<th>( u ) (mm)</th>
<th>( v ) (mm)</th>
<th>( w ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Means</strong></td>
<td>0.9984</td>
<td>-0.0005</td>
<td>-0.0162</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>0.0013</td>
<td>0.0009</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

**Results without disparity map**

<table>
<thead>
<tr>
<th>Statistics</th>
<th>( u ) (mm)</th>
<th>( v ) (mm)</th>
<th>( w ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Means</strong></td>
<td>0.9988</td>
<td>-0.0005</td>
<td>-0.0330</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>0.0019</td>
<td>0.0010</td>
<td>0.0015</td>
</tr>
</tbody>
</table>
Uniform flow field along x ($u'=1\,mm$) with angular misalignment $\alpha=-1^\circ$: correction of flow field using the \textit{disparity map}

\begin{itemize}
  \item \textbf{Without corrections}
  \begin{itemize}
    \item $u$ component of flow field
    \item Predictor: $u= 0.986\,mm$
  \end{itemize}
  \item \textbf{Corrected by means of disparity map}
\end{itemize}
Experimental results

Uniform flow field along $x$ ($u'=1\,mm$) with angular misalignment $\alpha=-1^\circ$:

correction of flow field using the *disparity map*

$v$ component of flow field

Predictor: $v=0\,mm$

*Without corrections*

*Corrected by means of disparity map*
Uniform flow field along $x$ ($u' = 1\, mm$) with angular misalignment $\alpha = -1^\circ$:

**correction of flow field using the disparity map**

- **Without corrections**
- **Corrected by means of disparity map**

$w$ component of flow field

**Predictor:** $w = -0.0169\, mm$
CONCLUSIONS

A correction to calculate viewing angles has been performed.

Misalignment errors between calibration plane and measurement plane have been analysed and a correction by means of disparity map has been proposed.

The proposed corrections, applied to simulated flow fields, allowed to achieve an improvement in results.